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The Transfer Theorem

M. Jarden

Tel-Aviv University

Abstract

The proof of the following theorem applies algebraic number theory, Galois theory, measure theory, elementary algebraic geometry, and model theory. It is one of the earlier theorems in "Field Arithmetic".

We consider a global field K, its absolute Galois group $\mathrm{Gal}(K)$, the Haar measure μ of $\mathrm{Gal}(K)$, and the first order theory $\mathcal{L}(\mathrm{ring}, K)$ of rings with a constant symbol for each element of K. For each $\sigma \in \mathrm{Gal}(K)$ let $\tilde{K}(\sigma)$ be the fixed field of σ in the algebraic closure $\tilde{K}$ of K. For each prime divisor $\mathfrak{p}$ of K let $\bar{K}_{\mathfrak{p}}$ be the residue field of K in $\mathfrak{p}$. If θ is a sentence of $\mathcal{L}(\mathrm{ring}, K)$, we write $A(\theta)$ for the set of all $\mathfrak{p}$ such that θ holds in $\bar{K}_{\mathfrak{p}}$ and $S(\theta)$ for the set of all $\sigma \in \mathrm{Gal}(K)$ such that θ holds in $\tilde{K}(\sigma)$. Finally, we use δ for the Dirichlet density of sets of primes of K.

Theorem: $\delta(A(\theta)) = \mu(S(\theta))$.

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