

1. Introduction to Lie groups and Lie algebras

- (a) What is a Lie algebra (abstract definition) and what are the basic examples?
What is a Lie group?
- (b) Exponential map. The case of a closed linear group.
- (c) Basic topology of Lie groups and their quotients. Covering spaces.
- (d) Complexification, automorphism groups.
- (e) Remarks on symmetric spaces/varieties.

2. Basic structure theory

- (a) Review of linear algebra.
- (b) Solvable, nilpotent, semisimple Lie algebras.
- (c) Combinatorics: Root space decomposition, root systems, Weyl group, root lattices, weight lattices.
- (d) Classification of semisimple Lie algebras.
- (e) Elementary representation theory; the case of $SL(2, \mathbb{C})$.

3. Representation theory

- (a) Universal enveloping algebra and the Poincare-Birkhoff-Witt Theorem.
- (b) Verma modules, infinitesimal characters, and a parametrization of irreducible representations.
- (c) The category \mathcal{O} and its Grothendieck group.
- (d) Harish-Chandra and Chevalley theorems on invariant polynomial functions.
- (e) Weyl character and dimension formulas, Kostant multiplicity formula.