istanbul matematiksel bilimler merkezi
istanbul center for mathematical sciences

Workshop on Nonlinear Dispersive PDE

long time dynamics, boundary value problems, integrability

Tuesday, July 12, 2016

09.30-10.20: Dario Bambusi (University of Milan) *Dynamics of a soliton in an external potential*

Consider the nonlinear Schrödinger equation

$$-i\psi_t = -\Delta\psi - \beta(|\psi|^2)\psi + \epsilon V\psi, \quad \beta \in C^\infty(\mathbb{R}), \quad V \in \mathcal{S};$$

it is well known that, when $\epsilon = 0$, under suitable conditions on β, the NLS admits traveling wave solutions (soliton for short). When $\epsilon \neq 0$, heuristic considerations suggest that the soliton should move as a particle subject to a mechanical force due to the potential. The problem of understanding if this is true or not has attracted a remarkable amount of work and it has been show that in the most favorable cases, the dynamics of the soliton is close to the dynamics of a mechanical particle at least for times of order$\epsilon^{-3/2}$. Numerical investigation, done in the case of a $V = \delta$ have shown that this is not true for longer times.

In will show that the orbit of the soliton remains close to the mechanical orbit of a particle for much longer times, namely for times of the order ϵ^{-r} for any r. The main point is that one has to renounce to control the position of the soliton on the orbit.

The proof is composed by three steps: introduction of Darboux coordinates, development of Hamiltonian perturbation theory and use of Strichartz estimates.

This is a joint work with Alberto Maspero.

10.20-10.50: *Discussions/Questions/Coffee Break*

10.50-11.40: Luc Molinet (François Rabelais University of Tours) *Improvement of the energy method for strongly non-resonant dispersive equations and applications*

We propose a new approach to prove the local well-posedness of the Cauchy problem associated with strongly non resonant dispersive equations. This approach combines the classical energy method with estimates in Bourgain's norms. It seems particularly efficient to prove unconditional well-posedness results both on the real line and on the torus. This is a joint work with Stéphane Vento (U. Paris 13).
As an example we obtain unconditional well-posedness of the Cauchy problem in the energy space for a large class of one-dimensional dispersive equations with a dispersion that is greater than the one of the Benjamin-Ono equation. At the level of dispersion of the Benjamin-Ono, we also prove the well-posedness in the energy space but without unconditional uniqueness. Since we do not use a gauge transform, this enables us in all cases to prove strong convergence results in the energy space for solutions of viscous versions of these equations towards the purely dispersive solutions. Finally, it is worth noticing that our method of proof works on the torus as well as on the real line.

11.40-12.00: *Discussions/Questions*

12.00-14.00: *Lunch*

14.00-14.50: Zaher Hani (Georgia Institute of Technology) *Long-time dynamics and turbulence of nonlinear waves*

Over the past twenty years, the long-time behavior of small amplitude solutions to nonlinear dispersive and wave equations on Euclidean spaces ($\mathbb{R}^n$) became relatively well-understood. In contrast, the situation is much less understood on bounded domains, that feature a markedly different and rich set of behaviors. In particular, the dynamics in this setting is characterized by out-of-equilibrium behavior, in the sense that solutions typically do not exhibit long-time stability near equilibrium configurations.

At the level of the physics underlying these problems, studying this out-of-equilibrium behavior leads to an interesting interplay between dynamics and statistical mechanics, in what is often known as wave turbulence theory. At the level of the mathematics, this study features an interaction between PDE methods, dynamical systems theory, probability theory, as well as a surprising and very elegant input from analytic number theory. In this talk, we shall discuss all these aspects, and survey some recent advances in this direction of research.

14.50-15.20: *Discussions/Questions/Coffee Break*

15.20-16.10: Beat Raphael Schaad (University of Kansas) *Smoothing results for integrable equations*

Consider the Schrödinger operator $L_q = -\partial_x^2 + q$, where $q(x)$ is a decaying potential. There is a scattering map S which assigns to q some spectral data of L_q. The remarkable point is that this map is invertible. This inverse scattering theory goes back to Faddeev. Furthermore there is the famous connection of inverse scattering to the Korteweg-de Vries equation (KdV) which goes back to Gardner, Green, Kruskal and Miura.
In a work with A. Maspero we showed with the help of this approach that the KdV flow map is one smoothing. More precisely, we showed that solutions of the KdV equation can be approximated by solutions of the linearized KdV equation up to an error which has one square integrable derivative more.
In the periodic setting we showed using different methods that the KdV flow is one smoothing as well. This was a joint work with P. Topalov and T. Kappeler. Similar results also hold true for the defocussing cubic nonlinear Schrödinger equation.

16.10-16.30: *Discussions/Questions*

Place : IMBM Seminar Room, Boğaziçi University South Campus
For more information please visit `http://www.imbm.org.tr/ndpde16/ndpde16.html`.

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