Dynamics of a soliton in an external potential

Dario Bambusi
Università degli Studi di Milano

Abstract

Consider the nonlinear Schrödinger equation

$$-i\psi_t = -\Delta\psi - \beta(|\psi|^2)\psi + \epsilon V\psi, \quad \beta \in C^\infty(\mathbb{R}), \quad V \in \mathcal{S};$$

it is well known that, when $\epsilon = 0$, under suitable conditions on β, the NLS admits traveling wave solutions (soliton for short). When $\epsilon \neq 0$, heuristic considerations suggest that the soliton should move as a particle subject to a mechanical force due to the potential. The problem of understanding if this is true or not has attracted a remarkable amount of work and it has been show that in the most favorable cases, the dynamics of the soliton is close to the dynamics of a mechanical particle at least for times of order $\epsilon^{-3/2}$. Numerical investigation, done in the case of a $V = \delta$ have shown that this is not true for longer times.

In will show that the orbit of the soliton remains close to the mechanical orbit of a particle for much longer times, namely for times of the order ϵ^{-r} for any r. The main point is that one has to renounce to control the position of the soliton on the orbit.

The proof is composed by three steps: introduction of Darboux coordinates, development of Hamiltonian perturbation theory and use of Strichartz estimates.

This is a joint work with Alberto Maspero.