Improvement of the energy method for strongly non-resonant dispersive equations and applications

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Abstract

We propose a new approach to prove the local well-posedness of the Cauchy problem associated with strongly non resonant dispersive equations. This approach combines the classical energy method with estimates in Bourgain's norms. It seems particularly efficient to prove unconditional well-posedness results both on the real line and on the torus. This is a joint work with Stéphane Vento (U. Paris 13).
As an example we obtain unconditional well-posedness of the Cauchy problem in the energy space for a large class of one-dimensional dispersive equations with a dispersion that is greater than the one of the Benjamin-Ono equation. At the level of dispersion of the Benjamin-Ono, we also prove the well-posedness in the energy space but without unconditional uniqueness. Since we do not use a gauge transform, this enables us in all cases to prove strong convergence results in the energy space for solutions of viscous versions of these equations towards the purely dispersive solutions. Finally, it is worth noticing that our method of proof works on the torus as well as on the real line.