Smoothing results for integrable equations

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Abstract

Consider the Schrödinger operator $L_q = -\partial_x^2 + q$, where $q(x)$ is a decaying potential. There is a scattering map S which assigns to q some spectral data of L_q. The remarkable point is that this map is invertible. This inverse scattering theory goes back to Faddeev. Furthermore there is the famous connection of inverse scattering to the Korteweg-de Vries equation (KdV) which goes back to Gardner, Green, Kruskal and Miura. In a work with A. Maspero we showed with the help of this approach that the KdV flow map is one smoothing. More precisely, we showed that solutions of the KdV equation can be approximated by solutions of the linearized KdV equation up to an error which has one square integrable derivative more.

In the periodic setting we showed using different methods that the KdV flow is one smoothing as well. This was a joint work with P. Topalov and T. Kappeler.

Similar results also hold true for the defocussing cubic nonlinear Schrödinger equation.