The Efficient Inversion Problem (numerization of Gelfand's theory)

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A short course of 5 lectures, 2 hours each (total volume: 10 hours in class). Audience: Master level (university 4th and 5th years).

Program

Lecture 1: Origins and the statement of the efficient inversion problem.
Condition numbers in matrix and numerical analysis. The Banach algebra setting and Bezout equations. Definition of the critical constants. Examples, and a discussion.

Lecture 2: Why there exists no constructive proof of Wiener's $1/f$ theorem?
A short history of the proofs of the $1/f$ theorem (Wiener, Gelfand, Calderon, P.Cohen, D.Newman). The critical constant of the Wiener algebra. Conclusison.

Lecture 3: Weighted Beurling-Sobolev algebras of smooth functions.
Definition of the weighted Beurling-Sobolev algebras. The well-posedness of the inversion problem in these algebras (scheme of the proof).

Lecture 4: The non-commutative case: condition numbers of large matrices obeing a prescribed functional calculus.
Kronecker's theorem and Van der Waerden's problem. Examples of functional calculi. Analytic capacities of finite sets. Solving the problem for Besov (and some other) algebras.

Lecture 5: Algebras of large matrices having prescribed spectrum.
Inversion closed algebras. Blaschke products and H^∞/BH^∞ quotient algebras. The Carleson Embedding Property, and Weak Embedding Property. Further developments and open questions.

REFERENCES

[1] I.Gelfand, D.Raikov, and G.Shilov, Commutative normed rings, NY, Chelsea, 1964.
[2] A.Zygmund, Trigonometric series, Vol.I and Vol.2, Cambridge, 1959.
[3] D.Newman, A simple proof of Wiener's $1/f$ theorem, Proc. Amer. Math. Soc., 48(1975), 264-265.
[4] N.Nikolski, In search for the invisible spectrum, Ann. Inst. Fourier (Grenoble), 49:6 (1999), 1925-1998.
[5] N.Nikolski, Condition numbers of large matrices, and analytic capacities, St.Petersburg Math. J., 17:4(2005), 125-180.